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ABSTRACT

Research into mathematical processes and their development is first viewed from the perspective of previous research that was directed towards the reading process. It is felt that the study of mathematics affords the experimental psychologist a well-specified domain of information with which to examine the full range of cognitive processes. Details are provided regarding the direction and nature of work with cognitive developmental theory. Then, discussion of the psychometric approach to the study of mathematical ability is found to provide a broad picture of distinct intellectual ability complemented by a numerical factor and a mathematical factor. Cognitive research is organized into two major categories, and the current emphasis of recent investigations is noted. The document closes with a presentation of four major conclusions drawn from this review. (MP)

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What the Study of Mathematical Ability Can Tell Us About Cognition and its Development

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What Research in Mathematics Contributes to Knowledge About Cognitive Development

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When cognitive psychologists began their experimental assault on the reading process, they hoped to learn enough about the basic human information processes to accomplish at least two goals:

1) to be able to remediate reading difficulties, and

2) to describe adequately the complex interaction among component processes at different stages of skill and development, thereby increasing our general knowledge of cognition.

Given the level of success achieved in the experimental analysis of reading, it is not surprising that cognitive and information-processing approaches have been taken in the study of mathematical processes and their development. Certainly, some of the cognitive psychologist's interest in the study of quantitative ability has to do with a general failure of our educational system to ensure adequate school achievement in mathematics. But, in addition, the study of mathematics affords the experimental psychologist a well-specified domain of information with which to examine perhaps the full range of human cognitive processes. As with reading, a better specification of the interaction of the component processes implicated in mathematical competence should foster improved educational programs.

Fundamental Issues

Whereas some would no doubt argue, and many have, that mathematical aptitude is no more than a reflection of general intellect, as in Spearman's g, others have suggested, either through introspection or empiricism, that there is a human mental faculty specific to mathematical ability. Is it that mathematical ability is an "outgrowth of man's intuition," as the 19th century mathematician Kroenecker asserted (cited in Choat, 1978)? Or does at least one

component of mathematical ability concern great insight into problem solution independent of logical deduction (Danzig, 1930, cited in Choat, 1978)? That is, is there a specific intellectual ability that determines success in mathematical problem solving independently of general reasoning ability (Furneaux & Rees, 1978; Rees, 1981)? And if this is the case, is there a unitary mathematical ability that determines performance from the earliest levels of arithmetic skill to higher level functioning in the areas of geometry, algebra, trigonometry, and calculus? Perhaps, as Spearman (1927) argued, it is merely a matter of convenience that we think of all "mathematics" as the subject of one large component of our intellect. As we shall demonstrate, the psychometrically-oriented psychologists and statisticians studying human intellect have addressed many of these questions, but have provided equivocal answers.

Cognitive Developmental Theory

Cognitive developmental psychologists have been concerned with very similar questions, even though the domain of number has been studied most often because it provides an excellent vehicle for studying conceptual development rather than because of any particular interest in mathematical ability itself. As Siegler and Robinson (1981) note in their recent work on the development of the child's concept of number, it is no longer feasible to view understanding of number as mastery of a single concept. Developmental research has shown that children possess many, perhaps related, concepts of number. In their own work, they attempt to use the principle of convergent operations in multiple tasks to construct models of young children's cognitive representations of

number. By so doing, they show that early mathematical skills develop in relative isolation from one another. Also, their distinction between representation and process proves useful in understanding the systematic, hierarchical nature of the development of numerical ability.

It is noteworthy that other recent work in cognitive development has been consistent with these notions. Piagetians have viewed the development of most concepts in terms of general, cognitive-structural changes, with development of specific skills the result of the application of more general structures. However, Gelman, in her work on number (Gelman & Gallistel, 1978; Starkey & Gelman, 1982), argues that the very young child possesses certain "localized rather than domain-wide" numerical abilities (e.g., counting strategies), and that development of these skills does not fit Piaget's notions of separable stages and clear distinctions between stages. Rather, she chooses to interpret the abilities of the young child in the domain of number as evidence for natural human intellectual abilities, analogous to language abilities, and sees development in terms of increasing complexity of concepts. This view is consistent with Fischer's (1980) notions of domain-specific skills that develop into more general understandings. Keil (1981) also views Gelman's work as consistent with his theory of cognitive universals: We, as humans, possess a special competence to operate upon numbers in specific, constrained ways. This could be taken as evidence for a specific numerical intellectual ability.

To what extent, therefore, is all of mathematical ability an outgrowth of this early, specific competence? If the cognitive development of mathematical skills is systematic and hierarchical in the

sense discussed by Siegler and Robinson (1981), then we may be able to understand much about the ultimate development of high-level cognitive processes by tracking the development of mathematical skills from the earliest concepts to the most advanced levels.

Psychometric Tradition

To adopt this view seems to require some evidence that we are dealing with a unique ability or at least an identifiable set of abilities that can be distinguished from other intellectual abilities. Although, as mentioned earlier, the psychometric studies have provided no definitive picture of the status of mathematical ability in human intellect, there has been a rather rich history of research dealing with these issues.

Correlational and factor analytic research on mathematical ability was given impetus by a series of early "introspective" studies in which experts freely speculated as to the nature of mathematical ability (see Werdelin, 1958). In general, many cognitive processes were believed to be necessary for mathematical proficiency— among them reasoning, memory, and spatial abilities were mentioned often.

Although early correlational studies were inadequate methodologically, there was apparent concensus that a mathematical ability existed independently of general intelligence as captured by Spearman's g. Initially, Spearman's (1927) own treatment of mathematical ability was restricted to the re-analysis of data compiled by Rogers (1918) and Collar (1920). He found a numerical factor over and above the g factor, and this factor captured the fundamental computation processes. In 1927 and again in 1950 (Spearman & Jones,

1950), he rejected the notion that higher mathematical ability, as in geometry, is related to numerical ability through anything but g. As a result, he paid little attention to higher mathematical ability in his work on intelligence.

But Spearman's was not the last word. Thurstone (1938) was instrumental in advancing the view that intelligence was a function of at least seven distinct types of ability. Among these, his "N" factor or numerical ability was the single primary ability with obvious relevance to mathematical ability. In essence, this ability was best tested with simple and speeded numerical computations, as in the addition or multiplication of pairs of single-digit numbers (Carroll, 1978).

So, even though Spearman and Thurstone conceptualized intelligence in very different ways, a numerical computation ability was included in both views but higher-mathematical ability was not discussed.

One of the most comprehensive factorial analyses of mathematical ability was that of the Swedish researcher Werdelin (1958). Rather than attempting to isolate a unique factor representing an underlying "trait" of mathematical ability, Werdelin accepted the importance of Spearman's g in mathematical problem solving and instead focused on components of mathematical ability and how they predicted school achievement. Of the factors isolated, three were found to be unambiguous. One of these was a numerical factor (N) and was independent of a verbal factor (V) and a visual factor (S). He did find

a general mathematical reasoning factor (R), but could not distinguish it completely from a g-type factor.

While reminiscent of other numerical factors, Werdelin's "N" factor was described also in terms more similar to modern concepts of the automaticity of overlearned routines. Coombs (1941) had concluded that numerical ability represents agility in handling well-established associations and Werdelin emphasized the importance of automatizing reasoning, especially with regard to numerical operations, for the purpose of economizing mathematical problem solving. He saw the numerical factor as capturing the ability to automatize mathematical reasoning through extensive practice. In this way, little if any conscious thought or mental effort would be required to apply mathematical rules or principles in numerical problem solving. Automaticity would be possible for any level of problem complexity, requiring more practice for greater complexity. As we shall see, automaticity is the area of concern of our own work and the aspect of mathematical problem solving that we think is essential in a developmental progression through mathematical skills of increasing complexity.

Concurrently with Werdelin's study, Wrigley (1958), in Britain, conducted both a critical survey of earlier work and an ambitious study of his own. In order to avoid criticisms levied against earlier studies, Wrigley carefully chose 4 mathematics achievement tests along with 11 other numerical, verbal, and spatial abilities tests. Thus, he was able to determine the status of mathematics ability relative to g as well as to some of the primary mental abilities. His results supported

a mathematical group factor distinct from both general ability and a numerical factor as well as from verbal ability.

Finally, we have noted that factorial studies of mathematical ability have been infrequent in the United States, possibly due in part to Spearman's suggestion that g accounts for much of advanced ability. Carroll and Maxwell (1979), in their comprehensive review of individual differences in cognition, referred to the Kit of Factor Reference Cognitive Tests (Harman & Ekstrom, 1976) and noted that it contained but a single factor concerned with number and quantity. This factor was defined, again, by speeded tests of simple arithmetic operations. There was an additional factor involving reasoning with quantitative concepts, but the factor was labeled "general reasoning." Carroll and Maxwell commented on the dearth of recent experimental analysis of these factors as well as the general lack of knowledge regarding individual differences in arithmetic skill development.

Summary. Thus, the psychometric approach to the study of math ability and intelligence has provided a broad picture of a distinct general intellectual ability complemented by two other factors: 1) a numerical factor concerned mostly with basic computational skills, and 2) a mathematical factor that incorporates advanced ability in mathematics relevant to school achievement.

Cognitive Research

Given the recent surge of interest in the development of math ability among cognitive and cognitive developmental researchers, we may ask what these approaches can tell us about the nature of this putative mathematical ability.

It may be useful to organize the cognitive research into two major categories:

1) Studies that describe procedures (or procedural knowledge) used by children when solving specific mathematical tasks. These include attempts to construct models of children's conceptual knowledge of different quantitative realtionships and to describe the interrelationships between conceptual and procedural knowledge in problem solving performance. We will refer to these studies as "problem solving" or "cognitive science" studies of mathematics.

2) Studies examining the general increase in mathematical problem solving efficiency and how this may account for the development of skill in mathematics. We will refer to these studies as "information processing" studies of mathematics.

Cognitive Science

Much of the cognitive research has concentrated on simple addition and subtraction computation problems, presented in numerical form as well as in word problems. As demonstrated in a recent volume of papers on addition and subtraction (Carpenter, Moser, & Romberg, 1982), heavy emphasis has been given to the development of strategies used by children who solve these problems, both correctly and incorrectly. There are many studies describing the counting strategies of very young children (e.g., Fuson, 1982; Steffe, Thompson, & Richards, 1982), but the most important contribution of cognitive psychology lies in the precision and clarity of description of the stimulus materials. Using concepts that have proven useful in the study of language and various complex problem solving tasks, the cognitive psychologist has been able to classify math problems in terms of their logical, semantic, and

syntactic attributes. Hence, children's performance on a range of problems can be evaluated in terms of operations that need to be applied to problems representing different quantitative relationships presented in different formats (Carpenter & Moser, 1982; Nesher, 1982). These elaborate descriptions have led some researchers to formulate models of the child's internal representations of computational procedures (procedural schemata) and of conceptual understandings (conceptual schemata). Riley, Greeno, and Heller (1982), Resnick (1982), and Vergnaud (1982) have all attended to the interaction of these cognitive structures in accounting for children's performance in simple arithmetic computation and word problems. Not only is the terminology used in these and related studies typical of modern theories of cognitive science and psychology, but some of the methods are, as well. For example, computer simulations of the arithmetic performance of young children have been employed to generate more sophisticated theories of the child's understanding of number (Brown & Vanlehn, 1982; Riley & Greeno, Note 1). Indeed, Romberg (1982) claimed that there is a general paradigm evolving in which researchers are constructing cognitive models of children's math problem solving appropriate for different developmental levels. From this perspective, development is viewed as contingent upon successful induction both of proper computational strategies and of concepts about number and numerical relationships. Development of adequate conceptual schemata will allow for the acquisition of more sophisticated strategies (procedures), and these in turn will allow for the increase in complexity of conceptual understanding. These notions stand in contrast to the past traditions in education, which have typically relied on acquisition of procedural

or conceptual knowledge, but not both. If the procedural knowledge is emphasized at the expense of the conceptual, then highly-skilled computational experts may develop. But, as Hatano (1982a) warned, these may be only "routine experts"— experts in terms of speed, accuracy, and automaticity of simple calculations. Without the concomitant training in conceptual knowledge, however, further achievement may not occur, and indeed this characterized the situation in the U.S. in the not-too-distant past. On the other hand, an overemphasis on conceptual knowledge, as in the "new math" programs of the recent past, would similarly be problematic. According to some, for example, Toyama (cited in Hatano, 1982b), computational ability is an important component of analytical ability, so that early competence in calculation ability may facilitate analytic problem solving ability in general.

Summary. This very brief and selective summary of cognitive problem solving approaches to the developmental study of math abilities suggests an important departure from the messages of the psychometric studies. The application of new theories and methodologies in cognitive science may be responsible for the view that math ability and its development are largely a function of the construction of knowledge schemata, both procedural and conceptual. The processes by which these schemata are formed initially and developed in time may be domain- or task-specific at first, but develop into more general knowledge structures, thereby allowing for general intellectual or problem solving strategies to appear.

Therefore, we might ask what the status of mathematical ability is, considered from this perspective. The work we have reviewed is

consistent with the earlier-stated modern views of cognitive development that speak of domain-specific skills developing into more generalized abilities. Yet, there does not seem to be any need to posit a specifically "mathematical" ability; only procedural knowledge involved in early computational ability and the conceptual knowledge involved in understanding various quantitative relations appear to be "mathematical" in nature, but we do not yet have a clear picture of their importance in the development of high-level ability in the mathematical domain.

Information Processing

Although there are examples in the cognitive science research of applications of information processing theory, typically the studies have fallen into the problem solving tradition of cognitive psychology. We feel that a complete specification of the development of math ability will require a consideration of the role of at least one set of information processing constructs— that having to do with the allocation of attention in a limited capacity system. As math problems become more complex, then solution may depend on more than just adequate procedural and conceptual knowledge: It is possible that efficient allocation of attention to various components of a task will be essential for optimal application of the knowledge schemata. Inefficient allocation can inhibit to varying degrees the application of available knowledge— hence, competence/performance disparities may be understood in terms of processing efficiency.

Case (1982) has argued that parallel developments across different content domains may be accounted for by general growth of central

processing capacity. Shavelson (1981) has drawn attention to Case's views and suggested that automatization of arithmetic operations, through extensive practice and overlearning, will reduce demands on memory and thereby allow for additional storage space for new incoming information. That is, the development of processing efficiency may be seen as an enabling condition for the acquisition of new information, whether procedural or conceptual. Such notions are consistent with the views of Coombs (1941), Werdelin (1958), and Hatano (1982a,b) mentioned earlier.

These concepts of automaticity may be useful in understanding the results of much of the information processing work in mathematical cognition. The bulk of this work has used chronometric analysis, or mathematical modelling of reaction times (RTs) to simple arithmetic problems in an attempt to describe implicit solution strategies. Beginning with the work of Groen and Parkman (1972), a number of studies have compared the RT patterns of subjects of different ages and found that very different models of strategies distinguish young children from older children and adults. In particular, the work of Ashcraft and colleagues (Ashcraft & Fierman, 1982) and our own work (Kaye, Hall, Post, & Dineen, Note 2) have shown that children's RT patterns in simple addition are consistent with the slow, effortful minimum addend (MIN) counting strategy before grade 3 but begin to shift thereafter to the adult pattern of rapid, effortless retrieval of stored number facts from long-term memory. Further development beyond late elementary school is believed to be a matter of increasing efficiency of retrieval of these number facts. Even the transition from counting to retrieval strategies most likely results from the extended practice afforded simple

arithmetic problems, but the gain in capacity accompanying such a strategy shift has not been directly measured nor has its role in subsequent math problem solving been explored. Our current research involves measurement of attentional allocation during simple arithmetic problem solving using dual-task techniques. We are trying to show that strategy changes are of functional importance since they result in additional processing capacity that can be used to accomplish more complex problem solving, as in proficient word problem solving. We already know from the work of Carpenter and Moser (1982) that basic computational strategies are related to word problem solving performance; we intend to show that this is in part due to processing efficiency. It is our belief that the systematic, hierarchical developments in mathematical ability alluded to by so many researchers may be better understood in terms of mechanisms of efficient capacity allocation.

Conclusions

In closing, we draw four main conclusions:

1) Although historically there have been two or three major psychological approaches to the study of mathematical ability, the research questions have been similar. Is there a basic or set of basic intellectual abilities devoted to the domain of mathematics? If so, is there systematic development of skills dependent upon early numerical ability?

2) The two major cognitive approaches, the problem solving or cognitive science and the information processing approaches, are not incompatible

but need to be applied together more often in the study of math ability.

3) Mathematics tasks provide a coherent domain of information with which to study cognitive behavior. This is an improvement over the use of artificial tasks constructed to study problem solving behavior and also over most attempts to model processing of natural language, which poses many difficulties in attempts to categorize syntax and semantics. Studies of the cognitive representations of number tell us about cognition itself, in that they provide us with a better description of the relationships between internal concepts and performance than we have had previously. More important, they suggest we can analyze performance on a wide variety of cognitive tasks, math or otherwise, in terms of a systematic, experience-dependent, shift in cognitive structures that organizes procedural and conceptual knowledge in a specific content domain. In the future, they are likely to provide insights into cross-domain performance, where schemata of more generalized sorts can be implemented along with domain-specific schemata to allow for complex and intelligent problem solving behavior.

4) Finally, we should return to some of the pursuits of the psychometricians. While the modern cognitive approaches using math tasks will no doubt continue to reveal the intricacies of cognition and its development in general, we should continue to be concerned with the essence of math ability in particular. We need to explore further the importance of early competence in number, which is perhaps indicative of a basic intellectual ability. We need to understand how this early capacity relates to higher level mathematical proficiencies. Furthermore, we should monitor the interaction between numerical

abilities and other cognitive abilities, such as spatial and imagery ability, as skill in geometry and other advanced domains is developed. To do so will provide important information about human cognition that will complement current cognitive studies of mathematics.

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